Effects of Eddy Current Induced Sextupole Moments in the Booster During Ramping

CLS DESIGN NOTE - 3.2.69.2 Rev. 0

(formerly 2.1.33)

Date: 2000-Jan-07

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REVISION HISTORY

Revision	***Date***	***Description***	***Author***
A	1999-Dec-21	Original Draft	J.C. Bergstrom L.O. Dallin
0	2000-Jan-07	Issued for use.	J.C. Bergstrom L.O. Dallin

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subject: **Effects of Eddy Current Induced Sextupole Moments in the Booster During Ramping.**

date: **December 21, 1999**

from: **J. C. Bergstrom and L. O. Dallin**

reference: 2.1.33

1. Introduction

The effect of eddy currents produced in the dipole vacuum chambers of the booster during ramping has been investigated. The eddy currents caused by ramping the booster dipole magnets produce an effective sextupole field superimposed on the dipole fields. This leads to changes in the chromaticities, particularly an increased value of the vertical chromaticity. Since there are no separate sextupole magnets to correct the sextupole content of the booster, the increased chromaticity will cause particles that are off energy to experience a tune shift. Large tune shifts can cause the particles to cross destructive resonances which may lead to loss of beam. The problem is most pronounced shortly after injection when the beam size and, especially, the energy spread are largest, and when the eddy current induced sextupole strength is near its maximum value.

Tracking simulations indicate that the maximum sextupole strength should not exceed $k_2 \approx 0.6$ to 1.0 m^{-3}, the precise value being conditional on beam matching criteria. Low induced sextupole values can be achieved by reducing the dipole chamber wall thickness, by changing the vacuum chamber aspect ratio or by using a relatively slow ramp rate at the beginning of the ramp cycle.

2. Induced Sextupole Moment

The time-varying magnetic field of the booster dipole magnets induces eddy currents in the metallic vacuum chamber. The magnetic field inside the vacuum chamber due to the eddy currents is given by[1]

$$B_y \;=\; 2\,F\,\mu_0\,\sigma \frac{h}{g}\,\frac{x^2}{2}\,\frac{\partial B}{\partial t} \quad ,$$

where the leading factor of 2 accounts for the top and bottom of the chamber, σ is the electrical conductivity, h and g are respectively the chamber wall thickness and chamber height and B is the magnetic field of the booster dipole magnet. An enhancement factor, F, determined by the chamber geometry (see below) has been included.

The field, B_y, varies as x^2 and thus corresponds to a sextupole field. The sextupole strength, or moment, is defined by

$$k_2 \;=\; \frac{1}{(B\rho)}\left(\frac{\partial^2 B}{\partial x^2}\right)_{x\,=\,0}$$

where $(B\rho)$ is the magnetic rigidity, the product of the field B and the radius of curvature, ρ. From the above equations we conclude

$$k_2 \;=\; 2\;F\;\mu_0\;\sigma\;\frac{h}{g\rho}\;\frac{1}{B}\;\frac{\partial B}{\partial t} \quad . \tag{1}$$

For an elliptical or circular vacuum chamber, the vertical thickness of the chamber wall increases with x, and this enhances the induced sextupole field. The enhancement factor, F, is solely a function of the elliptical chamber aspect ratio, g/w, where w is the chamber width, and is given by

$$F \;=\; 2\int_0^1 dx\left[x^2 + \left(\frac{g}{w}\right)^2 (1 - x^2)\right]^{1/2} \quad .$$

Thus, F = 1 when g << w, and F = 2 for a circular chamber.

The time dependence of the booster dipole field is of the form

$$B(t) \;=\; B_i + (B_e - B_i)\;f(t)$$

where the time dependent factor spans the range f(t) = 0 to 1, B_i is the field at injection and B_e is the field at extraction. The ratio of these fields is expressed by the parameter

$$\eta \;=\; \frac{B_i}{B_e}$$

and the logarithmic derivative in Eq. (1) may be written

$$\frac{1}{B}\frac{\partial B}{\partial t} = \frac{(1-\eta)}{\eta + (1-\eta)f}\frac{\partial f}{\partial t} \quad . \tag{2}$$

The parameter η is usually small. For example, injection at 0.23 GeV and extraction at 2.9 GeV corresponds to η = 0.08. In the limit $\eta \ll 1$, the time dependence of k_2 is approximately

$$k_2 \propto \frac{1}{\eta + f(t)}\frac{\partial f(t)}{\partial t} \ .$$

Since f(0) = 0, it is clear that k_2 will be minimized if

$$\left.\frac{\partial f(t)}{\partial t}\right|_{t=0} = 0 \ . \tag{3}$$

Now let us consider two ramping scenarios:

$$f(t) = \frac{1}{2}(1 - \cos(\omega t)) \tag{4}$$

$$\text{or} \quad f(t) = \left|\sin\left(\frac{1}{2}\omega t\right)\right| \ . \tag{5}$$

Scenario (4) is similar to a White circuit scheme, while (5) describes the LCR^2 resonant scheme originally proposed for the CLS. Note that the latter does not comply with Eq. (3).

For a representative estimate of k_2, consider a stainless steel vacuum chamber of height g = 32 mm and elliptical aspect ratio g/w = 0.5. The booster frequency is 2.5 Hz ($\omega=5\pi$), the radius of curvature of the booster magnets, ρ, is 7.25 m, $\sigma = 1.35\text{x}10^6\ \Omega^{-1-}m^{-1}$ (316 stainless) and η = 0.08. In Figure 1 and Figure 2, k_2 is shown as a function of the beam energy for the two ramping scenarios, for various wall thicknesses, h. The function k_2 is largest at low energy, where it will have the most deleterious effect since the beam will not be fully damped. Also, the maxima in k_2 differ by roughly a factor of 4 between the two ramping scenarios.

Beam stability in the booster is sensitive to the maximum value of k_2 and, simultaneously, to the quality of the beam matching between the injected beam and the Twiss parameters of the booster at the injection point.

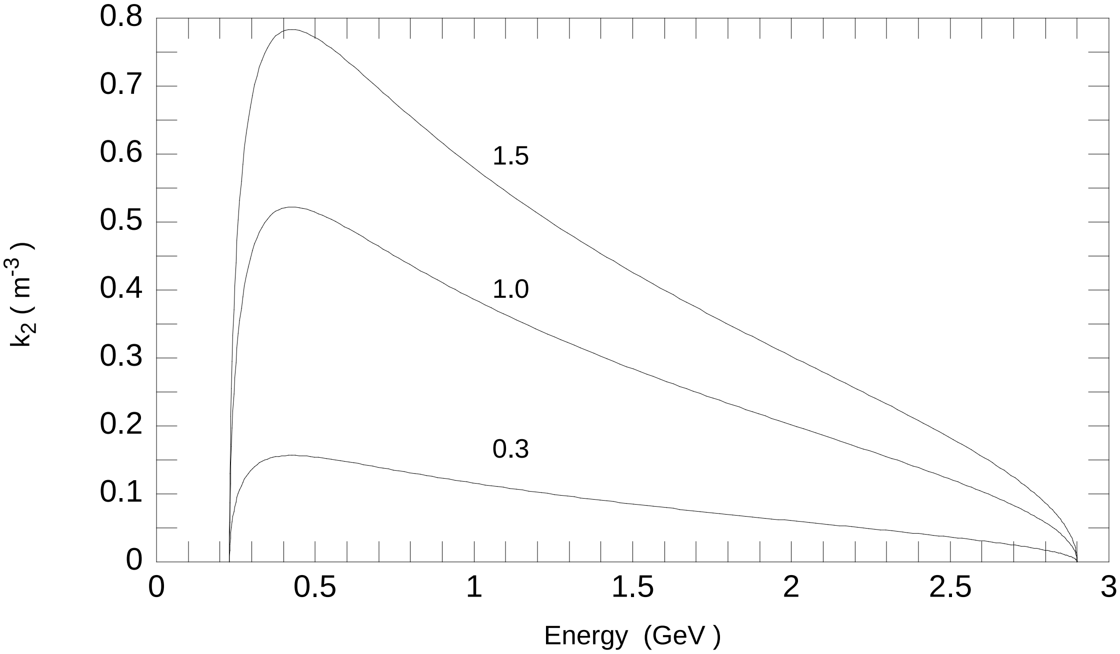

Figure 1 Induced sextupole moment for h = 0.3, 1.0 and 1.5 mm for the ramping function $f(t) = 1/2\ (1 - \cos(\omega t))$ over energies from 0.23 to 2.9 GeV.

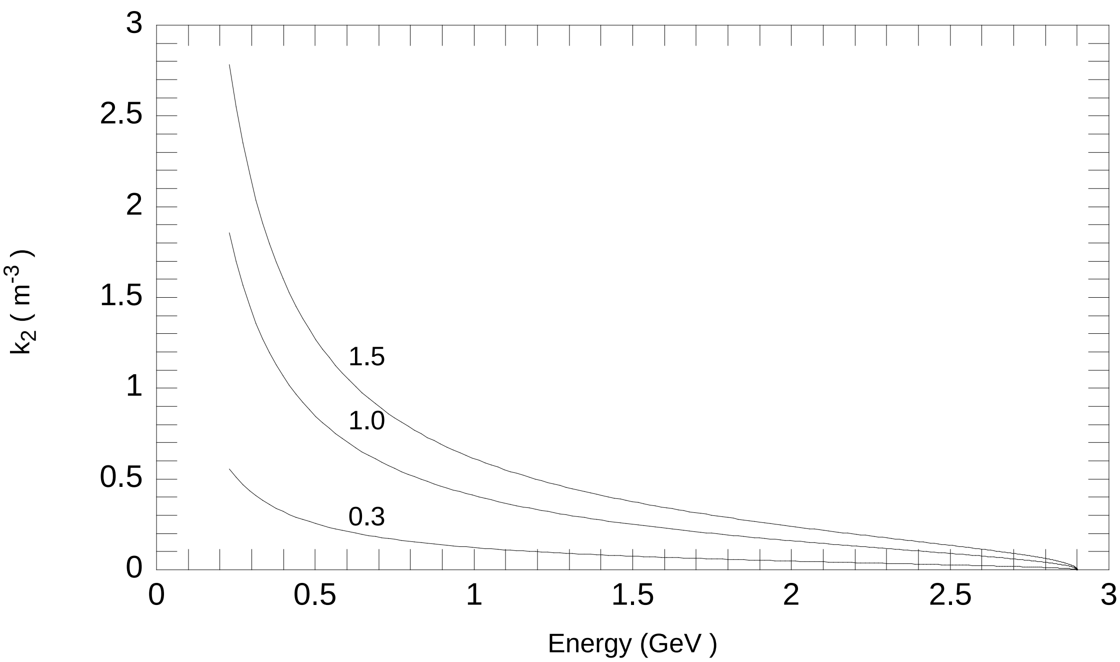

Figure 2 Induced sextupole moment for h = 0.3, 1.0 and 1.5 mm for the ramping function $f(t) = |\sin(\omega t/2)|$ over energies from 0.23 to 2.9 GeV.

3. Injected Beam

The machine functions at the injection point of the booster are:

α_x = 1.80, β_x = 3.97 m, η_x = 0.29 m, η'_x = - 0.05

α_y = - 2.71 and β_y = 14.53 m .

As well, the injected beam is assumed to have an energy spread of ±0.15% and a phase spread of 127 degrees at 500 MHz. (This phase spread corresponds to 720 degrees of phase, at the linac frequency of 2856 MHz, which always encompasses two linac bunches, but does include the phase spreading effect of the ECS. The energy spread is the maximum value expected from the ECS.) The emittance of the injected beam is assumed to be 300 nm-rad in both the horizontal and vertical planes. The large phase spread of the injected beam results in an effective energy spread of ±0.42% as shown in Figure 3 which shows the synchrotron motion after injection. This assumes a booster RF voltage of 0.16 MV at 0.23 GeV. Ideally, the beam functions of the injected beam should match the machine functions at the injection point. A perfect match may not be possible. Figures 4 and 5 show mismatched injected beams of ±3 σ extent in the vertical and horizontal planes, respectively, that would result in effective injected beam sizes of ±6σ. (Recall, $\sigma = (\varepsilon \beta)^{1/2}$.) The mismatches translate into a doubling of the effective beam size in the respective planes compared to a matched beam.

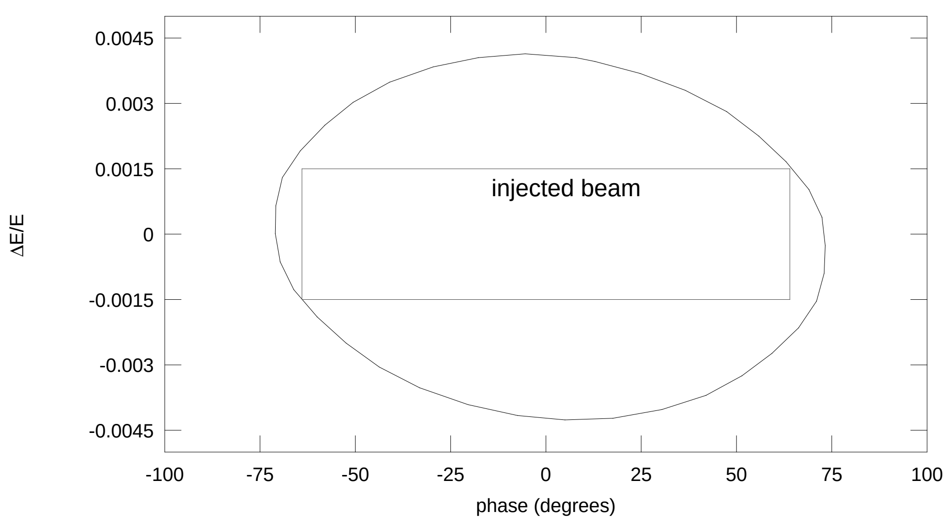

Figure 3 Longitudinal phase space of injected beam: The extreme corners of the injected beam (dashed line) have synchrotron motion shown by the solid curve.

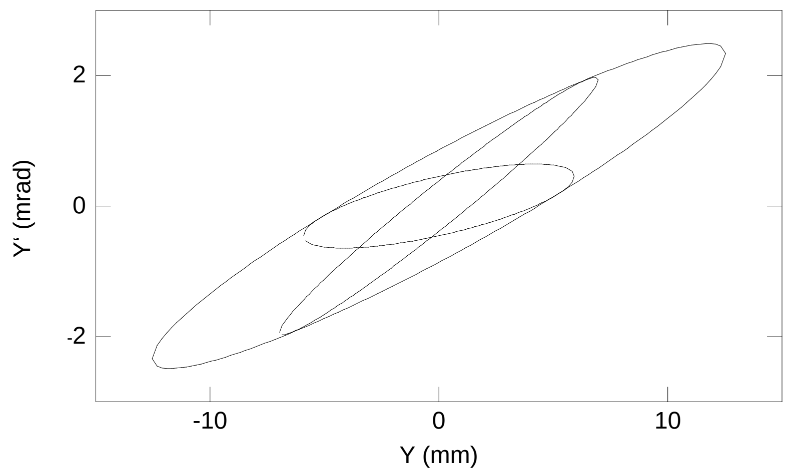

Figure 4 Vertical phase space: The mismatched beams have α = -1, β = 13 and α = -5, β = 18. They both represent $\pm 3\sigma$ of beam size and result in an effective beam size of $\pm 6\sigma$. (The small ellipses are the mismatched beams and the large ellipse is the resulting effective beam.)

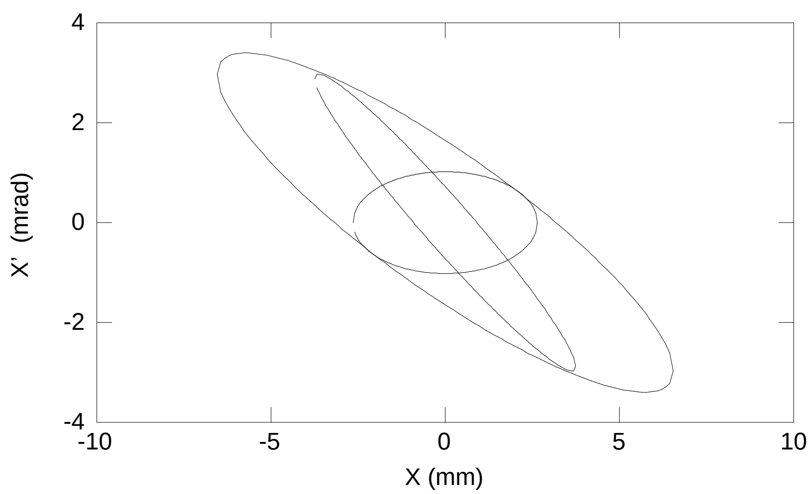

Figure 5 Horizontal phase space: The mismatched beams have α = 0, β = 2.6 and α = 4, β = 5.2. They both represent $\pm 3\sigma$ of beam size and result in an effective beam size of $\pm 6\sigma$.

4. Tracking with eddy current induced sextupoles

A single particle with initial coordinates corresponding to 6σ in both X and Y and with an energy deviation ($\Delta E/E$) of 0.42% was tracked to investigate the eddy current sextupole content in the booster dipole magnets when the injected beam is poorly matched to the booster. If the particle tracked for 50,000 turns without being lost the motion was considered to be stable. Table 1 shows the effects on beam size for a variety of k_2 values. Also shown are the resulting chromaticities, χ_x and χ_y, for the different values of k_2. Note that k_2 is assumed uniform over the magnetic length of each dipole. The beam sizes are calculated for the maximum β_x and β_y in the entire booster and also in the dipole where the vertical size is the largest. Excluding aperture restrictions, sextupole values up to $k_2 = 0.75$ can be tolerated. At this value, however, the maximum vertical beam would be larger than a dipole gap of 32 mm. To accommodate a mismatched beam and a dipole gap of 32 mm, we require $k_2 < 0.6$.

Table 1 Effect of Eddy Current Induced k_2 on a $\pm 6\sigma$ beam

k_2 m^{-3}	χ_x	χ_y	maximum X mm	maximum Y mm	maximum Y mm (dipole)
0.00	- 7.6	- 4.2	14.9	16.3	13.8
0.35	- 1.7	- 12.3	15.8	18.6	15.7
0.55	1.7	- 17.0	18.0	18.0	15.2
0.75	5.1	- 21.7	19.9	21.8	18.4
0.95	8.5	- 26.4	lost		particle

The above exercise was repeated for a single particle corresponding to $\pm 3\,\sigma$ in X and Y (i.e., a perfectly matched beam). A beam of $\pm 3\,\sigma$ would contain 95 % of the total beam, assuming a guassian distribution in the vertical amplitude. Table 2 shows the results for this case. For a matched (vertical) beam a dipole gap of 32 mm would be feasible.

Table 2 Effect of Eddy Current Induced k_2 on a $\pm 3\sigma$ beam

k_2 m^{-3}	χ_x	χ_y	maximum X mm	maximum Y mm	maximum Y mm (dipole)
0.00	- 7.6	- 4.2	8.3	8.2	6.9
0.55	1.7	- 17.0	8.7	8.8	7.4
0.75	5.1	- 21.7	9.3	12.7	8.4
0.95	8.5	- 26.4	10.2	11.0	9.3
1.05	10.2	- 28.1	11.2	13.2	11.2
1.15	11.9	- 31.1	lost		particle

5. Conclusion

The induced sextupole moment k_2 in the dipole in the dipole vacuum chambers can cause large shifts in the chromaticity. This, coupled with the increase in $\Delta E/E$, generated by the RF bucket to about ±0.4%, can seriously compromise the dynamic aperture. If the Twiss parameters of the injected beam are perfectly matched to the booster parameters, stability requires $k_2 < 1.0\ m^{-3}$. If the matching is poor, stability requires $k_2 < 0.6\ m^{-3}$ for a dipole gap of 32 mm. For a given vacuum chamber wall thickness, the White circuit wave form is preferred over the switched LCR resonant pattern since the respective maximum k_2 differ by nearly a factor of four.

6. References

1) D. A. Edwards and M. J. Syphers, “An Introduction to the Physics of High Energy Accelerators”, John Wiley and Sons, Inc., 1993.

2) N. G. Johnson and W. E. Norum, “Resonant Ramping Scheme for CLS Booster Dipole Magnets”, PAC ‘99, New York, p. 3764.